BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iaşi
Volumul 64 (68), Numărul 2, 2018
Secţia
CONSTRUCŢII DE MAŞINI

INSTANTANEOUS ANGULAR SPEED MONITORING BASED ON FIRST DERIVATIVE OF SIGNAL DELIVERED BY AN AC GENERATOR USED AS SENSOR

BY

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Received: May 29, 2018
Accepted for publication: August 31, 2018

Abstract. This paper proposes a theoretical and an experimental approach on computer aided monitoring of the instantaneous angular speed (IAS) for a rotating shaft using a n-poles AC generator as sensor (with a simple experimental setup consist of sensor, digital oscilloscope and computer). Basically the relationship between the amplitudes of the AC signal and its derivative is involved in IAS calculus. The IAS signal resources in rotating machine monitoring are revealed by some simple experiments on a lathe headstock and a comparison with two other IAS measurement procedures already proposed in a previous paper (Ciurdea and Horodincă, 2017b) was done.

Keywords: Instantaneous angular speed; sensor; signal derivative; data processing.

1. Introduction

This paper intends to improve some of our previous research results on computer aided instantaneous angular speed IAS monitoring, already revealed in a previous work (Ciurdea and Horodincă, 2017a; Ciurdea and Horodincă,

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2017b). The major part of the considerations from this previous papers related to the state-of-the-art and nomenclature (Ciurdea and Horodincă, 2017a) and especially on theoretical background as well as on experimental setup (Ciurdea and Horodincă, 2017b) are available here.

In the last previous work two different IAS measurement techniques were introduced in theoretical and experimental terms ($IAST_{app}$ and IASA), both based on computer aided processing of the signal delivered by an n-poles alternating voltage/current (AC) generator used as sensor, mechanically coupled on a rotary shaft. As a reasonable substitution for an ideal sensor, a two-phase stepper motor (Shinano Kenshi STH-56D101 with 200 steps per rotation) is used. When the rotor of stepper motor rotates at relatively high speed, its rotor generates two quasi-harmonic signals in phase quadrature (with $\pi/2$ phase shift between) as AC signals with 50 periods (n=50) on each rotation.

First technique introduced before (as $IAST_{app}$) is based on approximate measurement of the AC generator signal semi-periods (by computer aided zero-crossing moments detection) on a single phase. The second technique (as IASA) uses the measurement of the amplitude of the same signal and a proportional relationship between this amplitude and IAS. This last technique has an important disadvantage: the constant of proportionality between AC signal amplitude and IAS should be previously known.

This paper introduces a third IAS measurement technique based on the evolutions of amplitudes of numerical first derivative of the AC generator signal. All these three different techniques are available for both signals generated by sensor (each signal produced by a phase of the sensor).

This paper is organized as follows: in Section 2 a theoretical approach of this new IAS description method is done, Section 3 describes the experimental setup; Section 4 describes some experimental results with short comments; Section 5 is used for conclusions and future work.

2. IAS Measurement Based on Derivative of the AC Signal

As we already mentioned in (Ciurdea and Horodincă, 2017b) the instantaneous voltage $u(t)$ generated by an ideal AC generator if its rotor rotates with a constant instantaneous angular speed ω is given by the equation:

$$u(t) = U \sin(n \cdot \omega \cdot t) \tag{1}$$

The generator describes indirectly the IAS, actually included inside the sinus function argument. There is a first mathematical way to extract the IAS, as it follows:

$$\omega = \frac{1}{n \cdot t} \arcsin\left[\frac{u(t)}{U}\right] \quad \text{with} \quad u(\text{t}) < U \tag{2}$$

This way to find out the IAS is not privileged in this paper.

A second way supposes to use the first derivative of the voltage $u(t)$ related to time as:

$$u' = \frac{du(t)}{dt} = U_d \cdot \cos(n \cdot \omega \cdot t) = U \cdot n \cdot \omega \cdot \cos(n \cdot \omega \cdot t) \tag{3}$$

Of course this result of derivative is available only if the IAS ω is constant.

According to Eq. (1) and (3) the IAS can be simply defined as the ratio between two successive values of amplitudes, one of derivative of $u(t)$ and the next of $u(t)$ signals:

$$\omega = \frac{U_d}{n \cdot U} = \frac{U \cdot n \cdot \omega}{n \cdot U} \tag{4}$$

For n=50 poles, there are 50 periods of $u(t)$ signal (per rotation) or $2n$=100 semi-periods, so 100 values of amplitudes (or peak-values as well on each positive and negative cycle). This is also the value of sampling rate of IAS (samples per rotation or s.p.r.). If both signals delivered by the AC generator are used then the sampling ratio becomes $4n$=200 *s.p.r*.

The computer aided detection procedure of the amplitude (or peak value) between two consecutive zero-crossing moments of the signal, was plenty explained in (Ciurdea and Horodincă, 2017b).

Suppose that $u[i]$ and $u[i+1]$ are two consecutive generic numerical samples of $u(t)$ signal and $t[i]$, $t[i+1]$ are the sampling time of these samples. The first derivative of $u(t)$ signal can be numerically obtained in two different approximate procedures, both as an approximation of infinitesimal intervals involved in derivative definition (as the ratio du/dt) by finite intervals (as the ratio $\Delta u/\Delta t$). By this approximation, the slope of a local tangent line of the curve $u(t)$ (which is the result of derivative) is replaced (approximated) by the slope of a local secant line.

In a first procedure called one sided forward difference approximation of the derivative (Chapra and Canale, 2015) the intervals are defined as $\Delta u= u[i+1]- u[i]$ and $\Delta t= t[i+1]- t[i]$. A generic numerical sample of u' is defined as:

$$u'[j] = \frac{u[i+1] - u[i]}{t[i+1] - t[i]} \tag{5}$$

With the sampling time $t[j]$ conventionally defined as:

$$t[j] = \frac{t[i+1] + t[i]}{2} \tag{6}$$

A second procedure -central difference approximation of the derivative- according with (Chapra and Canale, 2015; Zhao *et al.*, 2018) allows a better numerical approximation of the derivative of $u(t)$ if three consecutive generic samples are considered: $u[i\text{-}1]$, $u[i]$ and $u[i+1]$ with the sampling times $t[i\text{-}1]$, $t[i]$ and $t[i+1]$. A generic numerical sample of *u'* is defined as:

$$u'[j] = \frac{u[i+1] - u[i-1]}{t[i+1] - t[i-1]} \tag{7}$$

With the sampling time $t[j]$ conventionally defined as:

$$t[j] = \frac{t[i+1] + t[i-1]}{2} \tag{8}$$

This second procedure will be used in this paper. The slope of the tangent line in the point $t[i]$, $u[i]$ is almost the same with the slope of secant line between the points $t[i\text{-}1]$, $u[i\text{-}1]$ and $t[i+1]$, $u[i+1]$ especially if $t[i+1]\text{-}t[i]= t[i]\text{-}t[i\text{-}1]$. In this situation of course $t_j=t_i$. If $u(t)$ is numerically described with n samples then $u'(t)$ is described with n-2 samples ($i_{min}=2$, $i_{max}=n\text{-}1$).

There is a logical problem here: certainly the procedure is useful especially when is expected to describe a variable IAS related to time, which means that in a real approach the definition of $u(t)$ voltage given in Eq. (1) should be rewritten as:

$$u(t) = U\sin(n\cdot\omega\cdot t) + \sum_{k=1}^{m} U_k \sin(n\cdot\omega_k\cdot t + \varphi_k) \tag{9}$$

The signal $u(t)$ is periodic. There is a dominant component (with angular frequency $n{\cdot}\omega$, ω being the average value of IAS, which also determines the period $T=2{\cdot}\pi/n{\cdot}\omega$ of signal delivered by AC generator) and some variable components describable as a sum of m harmonic components (second term in Eq. (9), much smaller than the dominant). This approach is confirmed by experimental results. The variable components are generated usually by torsional vibrations and malfunctions of mechanical parts of the rotating system, transmitted to the shaft where the AC generator is placed.

The first derivative of the voltage $u(t)$ related to time (from Eq. (9)) is mathematically described by:

$$u'(t) = U\cdot n\cdot\omega\cdot\cos(n\cdot\omega\cdot t) + \sum_{k=1}^{m} U_k\cdot n\cdot\omega_k\cdot\cos(n\cdot\omega_k\cdot t + \varphi_k) \tag{10}$$

Suppose that the peak value (close by amplitude) of the signal $u(t)$ - from Eq. (9)- on the current semi-period is $U^* \neq U$. Suppose a numerical conversion of $u'(t)$ with a generic sampling time $t[j]$ such that in Eq. (10)

$\cos(n\cdot\omega\cdot t[j])=1$. A generic sample of $u'(t)$ -written as $u'[j]$- divided by $U^*\cdot n$ produces this result:

$$\frac{u'[j]}{U^*\cdot n}=\frac{U\cdot n}{U^*\cdot n}\omega+\sum_{k=1}^{m}\frac{U_k}{U^*\cdot n}\cdot n\cdot\omega_k\cdot\cos(n\cdot\omega_k\cdot t[j]+\varphi_k) \tag{11}$$

With a relatively good approximation in Eq. (11) the generic sample $u'[j]$ can be considered equal with the peak value U'^* of the signal $u'(t)$ on the current period. If $\alpha=U/U^*$ and $\beta_k=U_k/U^*$ the Eq. (11) becomes:

$$\frac{U'^*}{U^*\cdot n}\approx\alpha\cdot\omega+\sum_{k=1}^{m}\beta_k\cdot\omega_k\cdot\cos(n\cdot\omega_k\cdot t[j]+\varphi_k) \tag{12}$$

Eq. (12) is a possible new description of the IAS as IASND (instantaneous angular speed based on numerical derivative) measurement method. The ratio between two consecutive measurable peak values of $u'(t)$ and $u(t)$ -in the left-hand member of the Eq. (12)- is involved in the definition of a sample $\omega[j]$ of an approximate description of IAS (the right-hand member of equation).

$$\omega[j]=\alpha\cdot\omega+\sum_{k=1}^{m}\beta_k\cdot\omega_k\cdot\cos(n\cdot\omega_k\cdot t[j]+\varphi_k)\approx\frac{U'^*}{U^*\cdot n} \tag{13}$$

This paper reveals in experimental terms the resources of IASND monitoring technique during the same experiments already done before in (Ciurdea and Horodincă, 2017b), with a comparison with the techniques already developed before (IAST_{app} and IASA). A high accuracy is ensured in IAS description with Eq. (13) if $\alpha\to 1$ and $\beta\to 0$.

3. Experimental Setup

All the considerations related to the experimental setup (sensor, data acquisition system, computer, the rotating system whose IAS should be monitored) previously done in (Ciurdea and Horodincă, 2017b) are fully valid in this paper. Some supplementary simple computer programs in Matlab have been developed for data processing in order to find out the numerical derivative of the voltage generated by AC generator, peak values of signals, etc.

4. Experimental Results and Discussion

4.1. IASND and IAST_{app} Experimental Evolutions in Time Domain

The experiment described in Fig. 3 from (Ciurdea and Horodincă, 2017b) -a transient regime of a lathe headstock mirrored in the evolution of IAS of the output main shaft- is described here in Fig. 1, using IAST_{app} (as a comparison item) and IASND measurement techniques.

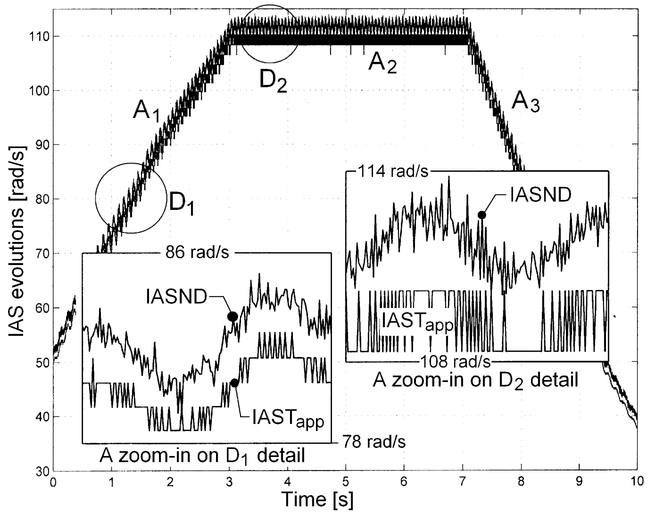

Fig. 1 – Experimental IAS evolutions ($IAST_{app}$ and IASND) on the output shaft lathe headstock in time domain.

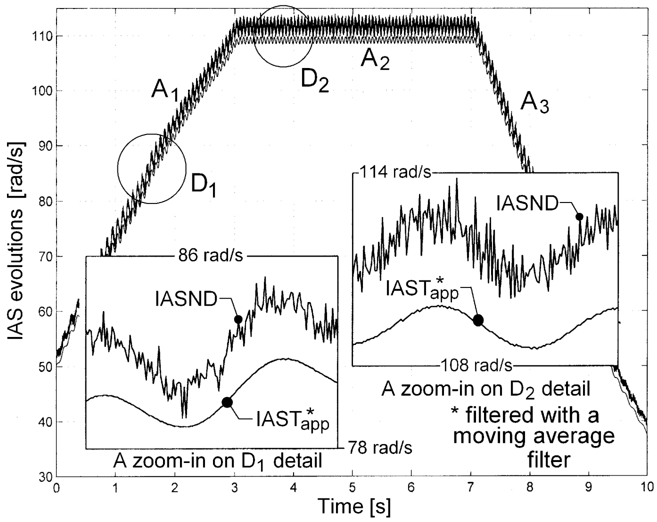

Fig. 2 – The evolutions from Fig. 1 with $IAST_{app}$ numerically filtered (moving average low-pass filter).

Fig. 2 present the same evolutions but with $IAST_{app}$ numerically low pass filtered (using a moving average filter). The filtering keeps only the low frequency parts of the signal $IAST_{app}$. It is not surprising that there is not a perfect fit between filtered $IAST_{app}$ and IASND. According with Fig. 3 (with both evolutions filtered), there is a relatively small offset (2.5 rad/s in D_2 detail, no shift of phase). The difference is due to the approximations made in Eqs. (12) and (13). Also it is possible that the $IAST_{app}$ measurement method (used here as reference) to be affected by some errors.

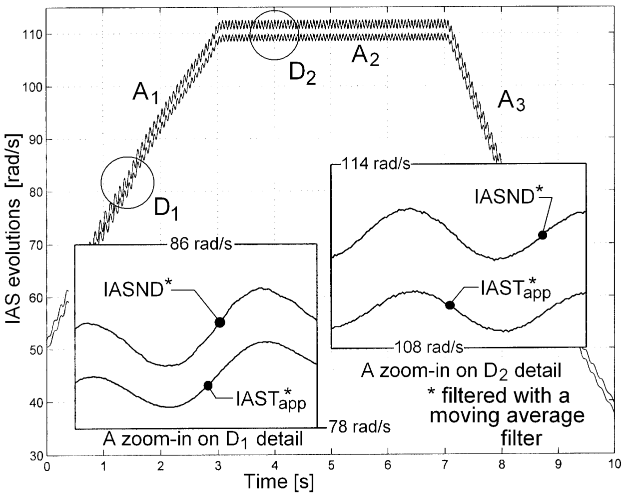

Fig. 3 – The evolutions from Fig. 1 with $IAST_{app}$ and IASND numerically filtered (low-pass filter).

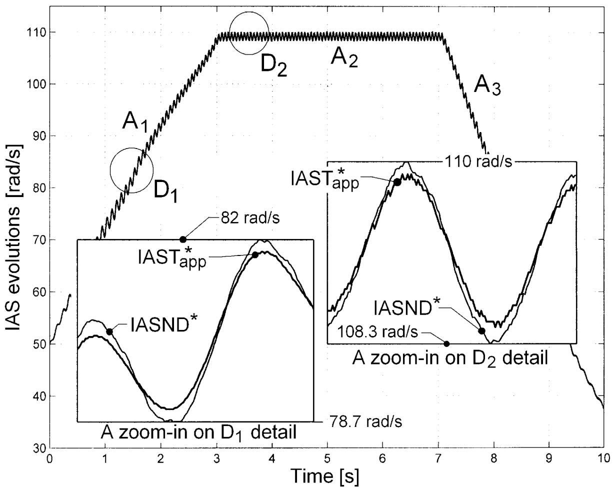

Fig. 4 – The evolutions from Fig. 3 with adjusted definition of filtered IASND evolution, according with Eq. (14).

It is interesting to remark that a correction of the Eq. (13) can be done in order to increase the fitting accuracy between filtered IAST and IASND as Eq. (14) indicates:

$$\omega[j] \approx 0.995595 \cdot \frac{U'^*}{U^* \cdot n} - 1.38381 \tag{14}$$

This relationship is generated as a result of numerically linear interpolation (curve-fit) of $IAST_{app}$ *versus* IASND (Ciurdea and Horodincă, 2017b). The result of this correction is exposed in Fig. 4. It is also possible to adjust the IASND evolution with a parabolic interpolation.

In Fig. 4 the amplitude of IASND is lightly bigger probably because of numerical derivation. This difference of amplitude (and shift of phase too) between filtered $IAST_{app}$ and IASND is also related with the filtering (moving average filter with different number of samples in the average). A systematic error is generated by the time delay (a quarter of period of the AC signal) between the amplitudes U^* and U'^* due to the numerical derivative.

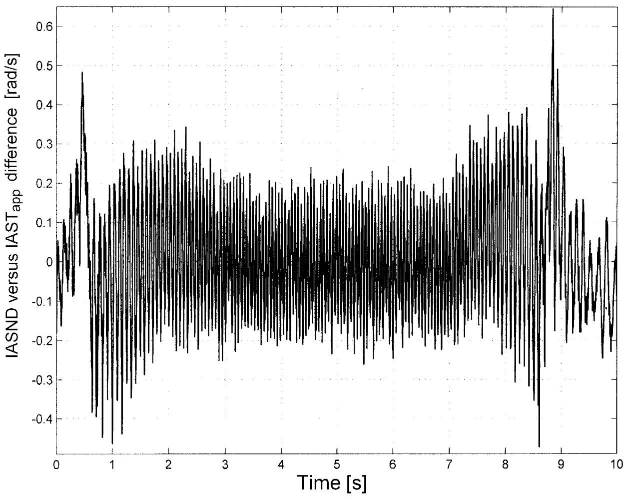

Fig. 5 – The evolutions of difference between IASND and $IAST_{app}$ related to Fig. 4.

However, despite some minor inconveniences, is confirmed that the IASND method seems to be a feasible approach in IAS monitoring. Fig. 5 presents the evolution of local difference between IASND and $IAST_{app}$ values (filtered evolutions). A maximum peak to peak difference less than 1.2 rad/s is revealed here.

4.2. IASND Experimental Evolutions in Frequency Domain

It has already been revealed previously (Ciurdea and Horodincă, 2017b) the resources of IAS evolutions (IASA and $IAST_{app}$) in frequency domain (disclosed by Fast Fourier Transform -FFT- of numerical signals). The IASND evolution also contains important information. Fig. 6 present the evolution of unfiltered IASND in frequency domain (with real amplitudes), in the same condition as those used for Fig. 12 from (Ciurdea and Horodincă, 2017b), related to the evolution on sequence A_2 from Fig. 2.

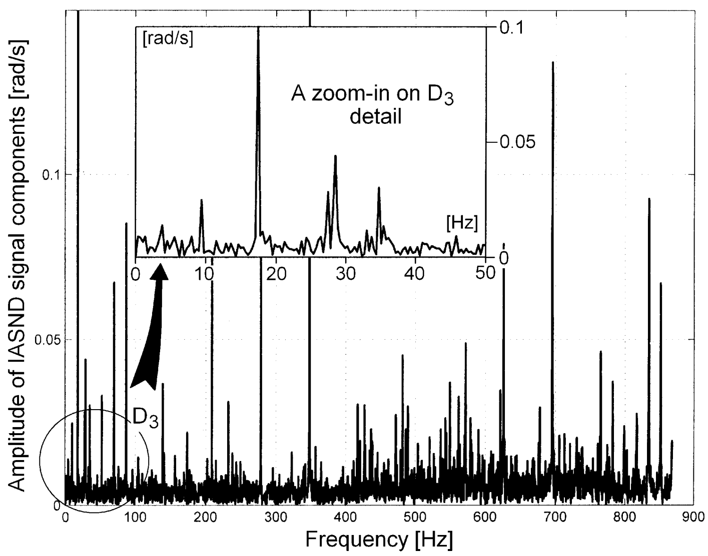

Fig. 6 – The evolution of IASND (on A_2 sequence from Fig. 2) in frequency domain by FFT.

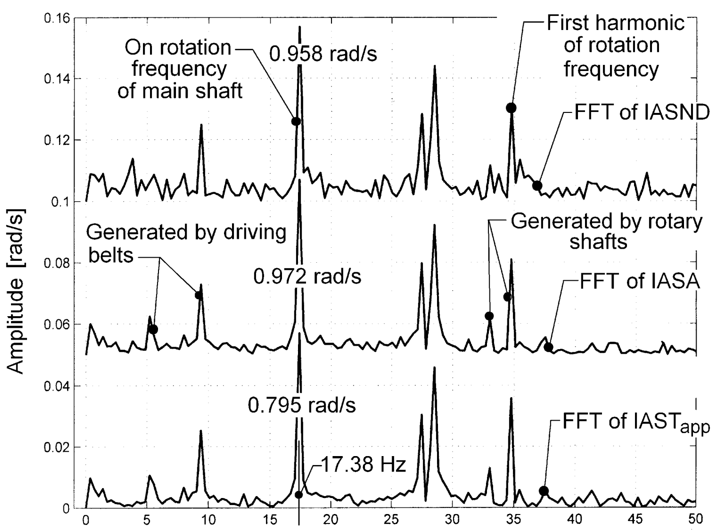

Fig. 7 – The evolutions in frequency domain. Spectra from 0 to 50 Hz for $IAST_{app}$, IASA (moved up with 0.05 rad/s) and IASND (moved up with 0.1 rad/s).

In theory there are some major differences between IASND and IASA spectra: the components with high frequency should be better described in IASND (the numerical derivative of *u*(*t*) signal increases the amplitudes of these components).

A comparison between the spectra shapes at low frequency (0 to 50 Hz) from $IAST_{app}$, IASA and IASND can be done related with the Fig. 7. The spectrum of IASA and IASND is moved up with 0.05 rad/s and 0.1 rad/s respectively. Here the peaks on 17.38 Hz frequency are graphically limited to value of 0.057 rad/s. The real amplitude of these peaks is written on Figure (0.958 rad/s on IASND, 0.972 rad/s on IASA and 0.795 rad/s on $IAST_{app}$).

As it is clearly indicated, there are not major differences in frequency domain. However, on Fig. 7 it seems that there is a better approach between $IAST_{app}$ and IASA spectra.

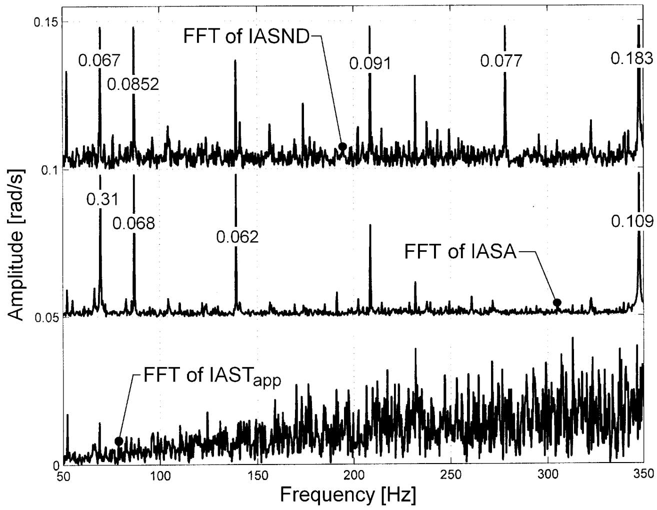

Fig. 8 – The evolutions in frequency domain. Spectra from 50 to 350 Hz for $IAST_{app}$, IASA (moved up with 0.05 rad/s) and IASND (moved up with 0.1 rad/s).

Fig. 8 presents the spectra of the three ways of describing the IAS (not filtered signals), between 50 and 350 Hz, using the same scale ratio. As already outlined in Fig. 7, the spectra of IASA and IASND are moved up with 0.05 rad/s and 0.1 rad/s respectively. Here also some peaks are graphically limited at 0.057 rad/s (on each of these peaks is written the real amplitude). It is clearly indicated that now there is a very bad description of periodical components on $IAST_{app}$. This is absolutely normal, taken into account the strong step variation of unfiltered $IAST_{app}$ evolution (depicted in Fig. 1).

In opposite, there are many elements of similarity between IASA and IASND, both evolutions in frequency domain describes almost the same periodical components. However the IASND evolution in frequency domain describes better the periodical components (some peaks from IASND are missing in IASA), because in IASND definition a numerical derivation is used. This should be the main advantage of IASND measuring technique when it is used for monitoring and diagnosis of rotary mechanical systems (Lei *et al.*, 2014) by FFT. If we suppose that the AC generator works perfectly then the periodical components from IASA and IASND signals (revealed by peaks in FFT) are strictly related with the lathe headstock condition (each periodical variation of internal or external mechanical loading of the lathe headstock generates periodical components of IAS). In reality some high frequency periodical components are generated by the generator itself. A significant part of the component with 17.38 Hz frequency (on rotation frequency of the main shaft, Fig. 7) is probably related with an imperfect clamping of the AC generator rotor in the main/output shaft of the headstock).

5. Conclusions and Future Work

As a continuation of the research results exposed in a previous paper (Ciurdea and Horodincă, 2017b), a third method of computer aided IAS monitoring is revealed in theoretical and experimental terms, using an n-poles AC generator as sensor. The signal delivered by generator is numerically derived. The ratio between the amplitudes of derived and underived signal is involved in an approximate definition of IAS as instantaneous angular speed based on numerical derivation (IASND). A small correction of IASND with a linear relationship was proposed.

The availability of this new technique was experimentally proved by signal processing analysis in time and frequency domain and by comparison with the results obtained using the previously defined methods of IAS monitoring (IASA and $IAST_{app}$).

This new measurement technique is expected to be useful in condition monitoring and diagnosis of mechanical rotary systems (by IAS evolution analysis in time and frequency domains). It is expected that many other available techniques of signal (Li *et al.*, 2017; Li and Zhang, 2017; Lei *et al.*, 2014) processing can be used for diagnosis based on IASND evolution.

In a future approach we expect that some research on computer aided identification of periodical components from IAS will be done. Some preliminary research proves that the identification is possible by computer aided curve fitting. In contrast to the FFT (which delivers only the frequency and the approximate value of amplitude for each periodical component) the curve fitting produces the amplitude (a better approximation), frequency and phase shift related to origin of time. This identification (and especially the phase shift) helps to find out the evolution of variable part of IAS (in time domain from identified fundamental and harmonics components) generated by a certain damaged part from the rotary mechanical system (*e.g.* a shaft, a driving belt, etc.). It is obvious that for a damaged rotary part the frequency of the fundamental component (induced in IAS) is the same with the frequency of rotation.

Acknowledgements. A part of the research results exposed in this paper were obtained thanks to the experimental resources provided by Developing a Research Platform for Efficient and Sustainable Energy (ENERED) Grant, POSCCE-A2-02.2.1-2009-4 ID 911, implemented at "Gheorghe Asachi" Technical University of Iasi Romania, funded by European Social Fund and Romanian Research Authority.

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MONITORIZAREA VITEZEI UNGHIULARE INSTANTANEE PE BAZA DERIVATEI SEMNALULUI FURNIZAT DE UN GENERATOR DE TENSIUNE ALTERNATIVĂ FOLOSIT CA SENZOR

(Rezumat)

Lucrarea introduce o metodă de monitorizare asistată de calculator a vitezei unghiulare instantanee (VUI), folosind semnalul generat de un generator de curent alternativ utilizat ca senzor. Se prezintă considerentele teoretice ale metodei şi se demonstrează experimental că VUI poate fi numeric descrisă cu o bună aproximaţie folosind raportul dintre amplitudinea semnalului derivat şi cea a semnalului nederivat şi o corecţie a acestui raport, pe baza unei relaţii de calcul liniare. Se foloseşte acelaşi echipament experimental şi se tratează datele provenite din aceleaşi experimente ca în lucrarea anterioară (Ciurdea and Horodincă, 2017b), fiind realizată o prezentare comparată cu bune rezultate a metodei propuse (pe evoluţii în domeniul timp şi în domeniul frecvenţă) cu cele două metode de descriere a VUI definite anterior.